Perfect Tracking of MIMO Systems using Dual Feedforward Method

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Abstract—We propose a new control architecture to perform Perfect Tracking for Multiple-Input Multiple-Output (MIMO) systems. Perfect Tracking is the task of very closely following a reference signal or trajectory by a system in the absence of modelling error or external noise. The architecture in this paper is based on the Dual Feedforward Method which has previously been applied for Single-Input Single-Output systems. Extension and generalization of the method to MIMO systems is non-trivial and our main contribution lies in designing the two feedforward paths through Right Matrix Fraction Description (RMFD) which enables perfect tracking of continuous-time MIMO systems. The paper presents the proposed architecture, the design methodology and illustrative simulation results. The architecture is also successfully implemented on the classic virtual Quadruple Tank laboratory apparatus.

Index Terms—Controller design, feedback, tracking control, modelling and simulation

I. INTRODUCTION

Perfect Tracking is the task where a system seeks to track a given reference trajectory with almost instantaneous settling time and deviating from the trajectory only very minutely, if at all. Tracking is an important and integral part of space-vehicle systems, robotic systems, manufacturing systems, and industrial operations involving control of temperature, pressure, flow, etc. These are dynamic systems, which suggest that the tracking mechanism in these systems needs to be dynamic; it should be able to track multiple reference points, if required, and achieve Perfect Tracking. Previously Perfect Tracking has been achieved in robot motion planning [1][2] and also in non-minimum phase magnetic levitation systems [3]. Another approach for perfect tracking by implementing internal model principle (IMP) [4][5] called Hybrid Reference Control (HRC) was developed in [6], further developed with optimal control [17] and using ANFIS [8].

In [9], a new method of achieving Perfect Tracking for non-minimum phase systems was introduced - The Dual Feedforward Method. However, a major limitation of most of these methods is that they are designed for SISO (Single Input Single Output) systems only. Most practical systems of interest are MIMO (Multiple Input and Multiple Output) systems, starting from home appliances to airplanes. Hence we look into the problem of designing a perfect tracking method for MIMO systems.

In this paper, it is proposed an architecture which will perform Perfect Tracking of MIMO systems using the Dual Feedforward Method. In the proposed model, the MIMO systems are considered as linear, continuous-time and square systems. The advantage of this architecture is that the adaptation is not taking place inside the feedback loop, which means internal stability is guaranteed while the controllers adapt in real-time, provided the two feedforward controllers stay stable. But the existing SISO-focused architecture cannot be directly implemented to MIMO systems. A new design method for constructing feedforward paths are proposed in this paper that makes this architecture suitable for MIMO systems.

The dual feedforward architecture can be implemented in a single controller. Alternatively, the dual feedforward can be separated in another different device with is linked to the feedback controller via transmission signal such as current or voltage signals. In modern control and automation technology, the transmission can be carried out using digital industrial communication such as RS 232, RS 485, ethernet or wireless communications, to illustrate [10][11][12][13]. The set-point implementation for perfect tracking with transient response improvement has been applied successfully on speed control of AC-motor by using ANFIS [8]. However, for implementation in a complex networked system, latency occurred in the communication network should be considered as it might induce big delay time [14].

The rest of the paper is organized as follows. Section II presents background materials, namely, MIMO system basics and the existing Dual Feedforward architecture for perfect tracking that works only on SISO systems. Section III presents our main contribution - the proposed architecture. Section IV discusses a couple of illustrative simulation results followed by presentation of the results of this method implemented on the classic problem of the Quadruple Tank laboratory apparatus. The paper concludes with a summary and an indication of some future directions.

II. BACKGROUND

Consider a linear MIMO system in state-space form as follows

\[ x(t) = Ax(t) + Bu(t) \] (1)

\[ y(t) = Cx(t) + Du(t) \] (2)

Here \( x(t) \) represents the state vector, \( A \) is the system matrix, \( B \) is the input matrix, \( C \) is the output matrix and \( D \) represents the feedback matrix. Let,
\[ u(t) \in \mathbb{R}^m \]
\[ y(t) \in \mathbb{R}^m \]

be input and output vectors respectively. As there are \( m \) inputs and \( m \) outputs, this is a square MIMO system.

### A. Transfer Function of MIMO Systems

Laplace Transformation of (1) and (2) leads to:
\[ sX(s) - x(0) = AX(s) + BU(s) \]
\[ Y(s) = CX(s) + DU(s) \]
where, \( \mathcal{L}\{u(t)\} = U(s) \), \( \mathcal{L}\{x(t)\} = X(s) \) and \( \mathcal{L}\{y(t)\} = Y(s) \). Assuming zero initial conditions, i.e., \( x(0) = 0 \), we obtain the following transfer function \( G(s) \):

\[
G(s) = \frac{Y(s)}{G(s)} = C(sI - A)^{-1}B + D
\]

Here, \( G(s) \) is of the following form:
\[
G(s) = 
\begin{bmatrix}
G_{11} & \cdots & G_{1m} \\
\vdots & \ddots & \vdots \\
G_{m1} & \cdots & G_{mm}
\end{bmatrix}
\]

where \( G_{ij}(s) \) denotes the transfer function from the \( i \)th input to the \( j \)th output.

### B. The Dual Feedforward Method for Perfect Tracking

For a SISO system, the Dual Feedforward architecture is given as follows (as proposed in [9]):

![Fig. 1. The Dual Feedforward Architecture](image)

Consider a SISO plant \( G(s) \):
\[
G(s) = \frac{N_{mp}(s)e^{st}}{D_{s}(s)D_{q}(s)}
\]

which is modeled by a proper Linear Time Invariant (LTI) system with non-minimum phase components such as time delays and right-half plane zeroes. \( G(s) \) can be split into two parts; a causal invertible part
\[
G_{i}(s) = \frac{K_{DCN_{mp}(s)}}{D_{s}(s)D_{q}(s)}
\]
and, a non-causal non-invertible part
\[
G_{noi}(s) = \frac{N_{mp}(s)e^{st}}{D_{s}(s)D_{q}(s)}
\]

So we have the following:
\[
G(s) = G_{i}(s)G_{noi}(s)
\]

As equation (8) is invertible, therefore it can be said that:
\[
G^{-1}_{i}(s) = \frac{D_{s}(s)D_{q}(s)}{K_{DCN_{mp}(s)}}
\]

Observing the Dual Feedforward Architecture there are two feedforward paths \( FF1(s) \) and \( FF2(s) \), where,
\[
FF1(s) = P_{des}(s)G_{noi}(s)
\]
and,
\[
FF2(s) = P_{des}(s)G_{i}(s)
\]

In order to guarantee perfect tracking the following conditions must be satisfied:
- \( FF1(s) \) and \( FF2(s) \) must be stable proper transfer functions.
- The designing parameter \( P_{des} \) and \( G_{noi} \) should have the following property: \( P_{des}(0)G_{noi}(0) = 1 \)

Implementation of this perfect tracking method cannot easily be extended to MIMO systems because in a MIMO system there can be multiple transfer functions defining the system. Therefore, we cannot split the system into two parts as easily we did for a SISO system. In the next section we present our new architecture for achieving perfect tracking of MIMO systems, where the MIMO system is decomposed using a different technique.

### III. MIMO System Tracking Using Dual-Feedforward Method

According to the Dual Feedforward architecture presented in Fig. 1, the system \( G(s) \) needs to be divided into two parts to make the feedforward paths. When handling a MIMO system such as in (6), it can be observed that there are more than one rational function to be split and hence we simply cannot execute the method which was applied for SISO. Nevertheless, if we wanted to execute similar method of splitting as used in a SISO system, we had to deal every transfer function in the MIMO system separately, which means the execution time for perfect tracking will be really long and not effective. To avoid these dilemmas, a different approach for splitting a MIMO system was used, which is called Matrix Fraction Description (MFD).

#### A. MFD (Matrix Fraction Description)

The Rational Matrix \( G(s) \) which was our transfer function, can be written as a fraction of two polynomial matrices. And, as the product of two matrices is not commutative, there are two different ways to proceed [15].

There exists a right matrix fraction description or RMFD which can be defined as:
\[
G(s) = G_{N}(s)G_{D}^{-1}(s)
\]

where the non-singular matrix \( G_{N} \) enters \( G(s) \) from th right. Here, non-singularity means that the determinant of the matrix is not identically zero. Similarly, the left matrix fraction description or LMFD of \( G(s) \) can be defined as:
Here, the non-singular matrix is entering $G(s)$ from the left. $G(s)$ can also be presented as:

$$G(s) = N(s)D(s)$$

where $N(s)$ is the numerator matrix polynomial function and $d(s)$ is the least common multiple of the denominators of the entries in $G(s)$. Using $N(s)$ and $d(s)$, RMFD can be formed as:

$$G(s) = G_N(s)G_D^{-1}(s)$$

where $G_D(s) = d(s)I$.

Here, $G_D(s)$ is not a proper transfer function; therefore constructing an unstable system in a real life situation is not feasible. So, to have another feedforward path we will use $G^{-1}_N(s)$ to give us the second feedforward path.

We now present the steps to establish the Dual Feedforward Method for a MIMO system:

1. The system is decomposed into its Right Matrix Fraction Description (RMFD), to establish the two feedforward paths. Performing RMFD will give us two functions $G_N(s)$ and $G_D(s)$.
2. $FF1(s)$ will be constructed as such that:
   $$FF1(s) = P_{des}(s)G^{-1}_N(s)$$
3. Then $FF2(s)$ would be constructed as such that:
   $$FF2(s) = P_{des}(s)G^{-1}_D(s)$$

While performing the Dual Feedforward Method for a MIMO system, it is assumed that the transfer function of the system can be split into RMFDs, considering they follow the coprime factorization property presented in [3]. To construct $P_{des}(s)$ we use the equation of the first feedforward path $FF1(s)$ considering the initial condition $s = 0$. So, the equation for finding $P_{des}(0)$ becomes:

$$P_{des}(0) = IG^{-1}_N(0)$$

where,

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The architecture presented here is based on feedforward paths but the feedback controller $K(s)$ plays a significant role, as it helps reduce the steady state errors in the system. And, finally the transfer function of the system becomes:

$$\frac{Y}{R} = G(s)(G^{-1}_D(s)P_{des}(s) + G^{-1}_N(s)P_{des}(s)K(s))/(1 + G(s)K(s))$$

IV. SIMULATIONS

In this section simulation of two systems are presented, where the Dual Feedforward method is applied to achieve perfect tracking. One is a 2-input, 2-output system, while the other is a 3 by 3 system.

A. Simulation-1

Consider a 2 by 2 system, whose transfer function is given as:

$$G(s) = \begin{bmatrix} 4 & 0.5(s+2) \\ s+2 & (s+1)(s+2) \end{bmatrix}$$

$$\begin{bmatrix} 0 & 8 \\ (s+1)(s+2) & 8 \end{bmatrix}$$

Following (16) and (17) the RMFD of $G(s)$ is:

$$G_N(s) = \begin{bmatrix} 4 & -0.5(s+2) \\ (s+1) & 2 \end{bmatrix}$$

and,

$$G_D(s) = \begin{bmatrix} (s+1)(s+2) & 0 \\ 0 & (s+1)(s+2) \end{bmatrix}$$

According to the (18) and (19), we need to find $G^{-1}_D(s)$, and $G^{-1}_N(s)$ to create out feedforward paths $FF1(s)$ and $FF2(s)$. Using (23) and (24) we get,

$$G^{-1}_N(s) = \begin{bmatrix} s+2 \\ (s^2+3s+18) & -2s-2 \\ (s^2+3s+18) & 8 \\ (s^2+3s+18) & 8 \end{bmatrix}$$

$$G^{-1}_D(s) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ (s+1)(s+2) \end{bmatrix}$$

After finding $G^{-1}_N(s)$ and $G^{-1}_D(s)$, $P_{des}(s)$ needs to be designed. To design $P_{des}(s)$ we have to keep in mind the condition of perfect tracking presented in (20).

With $s = 0$ in $G_N(s)$, we get

$$G_N(0) = \begin{bmatrix} 4 & -2 \\ 2 & 8 \end{bmatrix}$$

Altering the equation $P_{des}(0)G^{-1}_N(0) = 1$ gives us

$$P_{des}(0) = IG_N(0)$$

which means for this particular problem,

$$P_{des}(0) = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}$$
Thus, based on our $P_{des}(0)$ we can now design $P_{des}(s)$. For this particular $G(s)$, $P_{des}(s)$ has been designed as below:

$$P_{des}(s) = \begin{bmatrix} \frac{4}{(s+1)^2} & \frac{5}{s+1} & 0 \\ \frac{1}{s+2} & \frac{2}{s+1} & 0 \\ \frac{s}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} & \frac{s}{s+1} \end{bmatrix}$$

(28)

But before the execution of the Dual Feedforward Architecture on $G(s)$ let’s have a look at how $G(s)$ behaves when it was asked to track the value 1 at Output 1 and 2. From Fig. 3 it can be seen that none of the outputs could track the desired value of 1. Output 1 tracked a value of 4.5 and Output 2 tracked the value of 1.5. But observing the effect of Dual Feedforward architecture as shown in Fig. 4, it can be seen that both the outputs have tracked the desired value of 2.

Following (16) and (17) the inverse of the RMFDs become,

$$G^{-1}_D (s) = \begin{bmatrix} \frac{1}{(s^2+3s+2)} & 0 & 0 \\ 0 & \frac{1}{(s^2+3s+2)} & 0 \\ 0 & 0 & \frac{1}{(s^2+3s+2)} \end{bmatrix}$$

(30)

$$G^{-1}_N (s) = \begin{bmatrix} -0.4 & 0.2s+0.2 & 0.2s^2+0.2s \\ \frac{s+2}{(s^2+3s+0.4)} & -0.8 & \frac{1}{(s^2+3s+0.4)} \\ 0 & 0 & \frac{1}{s} \end{bmatrix}$$

(31)

Design a $P_{des}(s)$ as follows:

$$P_{des}(s) = \begin{bmatrix} 100s^2 & 0 & 0 \\ 0 & 100s^2 & 0 \\ 0 & 0 & 100s^2 \end{bmatrix}$$

(32)

When asked to track the value 2 at all outputs, the system behaves as in Fig. 5 without the Dual Feedforward method. As it is observed here, all the outputs of the system fails to track their reference value 2. The outputs track values of 3, 6 and 9 respectively. From Fig. 6 we can see that the reference value 2 has been perfectly tracked after the Dual Feedforward method has been applied.
V. AN APPLICATION: CONTROLLING WATER FLOW IN A QUADRUPLE TANK

In this section we apply our technique on The Quadruple Tank which is a classic problem and evaluate our results. In [16] the transfer function of the Quadruple Tank was given as:

\[
G(s) = \frac{3.7x_1}{62s+1} \frac{3.7(1-x_2)}{(30s+1)(90s+1)} \frac{23s+1}{62s+1} \frac{4.7x_2}{(90s+1)}
\]

where \((x_1 \text{ and } x_2)\) represent the proportion of the flow from the pump that goes into tanks. The system here has two multivariable zeroes which satisfy \(\text{det}(G(s)) = 0;\)

\[
(23s+1)(30s+1) - n = 0
\]

where

\[
n = \frac{(1-x_1)(1-x_2)}{x_1x_2}
\]

To make the system minimum phase, the value of \(x_1 + x_2\) should be greater than 1 but less than 2. Therefore, for our simulation we have taken \(x_1 = 1\) and \(x_2 = 0.5\), so \(G(s)\) becomes:

\[
G(s) = \begin{bmatrix}
\frac{3.7x_1}{62s+1} & \frac{3.7(1-x_2)}{(23s+1)(62s+1)} \\
\frac{0.85}{(90s+1)} & \frac{0.15}{(90s+1)}
\end{bmatrix}
\]

When the desired output \(u_1 = 1\) and \(u_2 = 1\) is injected into the system, \(G(s)\) fails to track the inputs, resulting in the output \(y_1\) and \(y_2\) reaching a different level as shown in the Fig. 8. Now, to apply the Dual Feedforward Method on this system, we need to find the RMFDs of the system and their inverses. These are found to be as follows:

\[
G_N(s) = \begin{bmatrix}
0.4(s+1) & 0.2 \\
-0.2 & 0.4(s+1)
\end{bmatrix}
\]

and,

\[
G_D(s) = \begin{bmatrix}
0 & s^2 + 1.6s + 0.8 \\
0 & s^2 + 1.6s + 0.8
\end{bmatrix}
\]

Using (35) and (36),

\[
G^{-1}_N(s) = \begin{bmatrix}
\frac{2.5(s+1)}{s^2+2s+1.25} & \frac{-1.25}{s^2+2s+1.25} \\
\frac{1}{s^2+2s+1.25} & \frac{2.5(s+1)}{s^2+2s+1.25}
\end{bmatrix}
\]

\[
G^{-1}_D(s) = \begin{bmatrix}
\frac{1}{s^2+1.6s+0.8} & 0 \\
0 & \frac{1}{s^2+1.6s+0.8}
\end{bmatrix}
\]

Fulfilling the condition \(P_{\text{des}}(0)G^{-1}_N(0) = I\), here \(P_{\text{des}}(s)\) is constructed as:

\[
P_{\text{des}}(s) = \begin{bmatrix}
\frac{0.4}{(s+1)^2} & \frac{0.2}{(s+1)^2} \\
\frac{-0.2}{(s+1)^2} & \frac{0.4}{(s+1)^2}
\end{bmatrix}
\]

Using (38), (37) and (39) we can construct our feedforward paths which will lead to the desired results as shown in Fig. 9.
VI. CONCLUSION AND FUTURE DIRECTIONS

A methodology for achieving perfect tracking on MIMO plants was presented based on using two feedforward controllers that share information between them. The accuracy of the proposed model was tested here on two square matrices, where one was a two-by-two system, the other a three-by-three system. The proposed architecture was then applied on the Quadruple tank laboratory apparatus, where it could successfully achieve Perfect Tracking of fluid level.

In the work presented here, $P_{\text{des}}(s)$ was designed slightly arbitrarily according to the system’s requirement but it need not be designed in this manner; the Ziegler-Nichols and Optimal Control methods can be used to design $P_{\text{des}}(s)$ more systematically. Although $K(s)$ is attached to the internal loop of the architecture but methods like Ziegler-Nichols can be used to design this feedback controller too. Areas of improvement for our current work are as follows. First, the method used to split the MIMO system is inefficient for a system with large number of inputs and outputs, as finding inverses of a large system is computation-intensive and time consuming. In addition, all the systems that were used were continuous-time and square, therefore a more generic architecture needs to be developed which can take care of discrete systems with unequal number of inputs and outputs. And last but not the least, a more generic structure of the design parameter $P_{\text{des}}$ and $K$ can be constructed.

REFERENCES


